



CANDIDATE
NAME

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9231/13

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

1 The cubic equation $7x^3 + 3x^2 + 5x + 1 = 0$ has roots α, β, γ .

(a) Find a cubic equation whose roots are $\alpha^{-1}, \beta^{-1}, \gamma^{-1}$. [3]

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(b) Find the value of $\alpha^{-2} + \beta^{-2} + \gamma^{-2}$. [2]

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(c) Find the value of $\alpha^{-3} + \beta^{-3} + \gamma^{-3}$. [2]

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2 The sequence $u_1, u_2, u_3, \dots$ is such that $u_1 = 1$ and $u_{n+1} = 2u_n + 1$ for $n \geq 1$.

(a) Prove by induction that $u_n = 2^n - 1$ for all positive integers n . [5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Deduce that u_{2^n} is divisible by u_n for $n \geq 1$. [2]

[illegible]

3 Let $S_n = 2^2 + 6^2 + 10^2 + \dots + (4n-2)^2$.

(a) Use standard results from the List of Formulae (MF19) to show that $S_n = \frac{4}{3}n(4n^2 - 1)$. [4]

[illegible]

- (b) Express $\frac{n}{S_n}$ in partial fractions and find $\sum_{n=1}^N \frac{n}{S_n}$ in terms of N . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings present.

- (c) Deduce the value of $\sum_{n=1}^{\infty} \frac{n}{S_n}$. [1]

4 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} k & 0 & 2 \\ 0 & -1 & -1 \\ 1 & 1 & -k \end{pmatrix},$$

where k is a real constant.

(a) Show that $\mathbf{A}$ is non-singular.

[3]

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The matrices $\mathbf{B}$ and $\mathbf{C}$ are given by

$$\mathbf{B} = \begin{pmatrix} 0 & -3 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} -3 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

It is given that $\mathbf{CAB} = \begin{pmatrix} -2 & -\frac{3}{2} \\ -1 & -\frac{3}{2} \end{pmatrix}.$

(b) Find the value of k .

[3]

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- (c) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by **CAB**. [5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

- 5 The curve C has polar equation $r = a \tan \theta$, where a is a positive constant and $0 \leq \theta \leq \frac{1}{4}\pi$.

- (a) Sketch C and state the greatest distance of a point on C from the pole. [2]

- (b) Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{4}\pi$. [4]

This image shows a full page of a worksheet designed for handwriting practice. It features ten sets of horizontal dashed lines, each set consisting of two parallel dotted lines. These lines are evenly spaced across the entire page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- (c) Show that C has Cartesian equation $y = \frac{x^2}{\sqrt{a^2 - x^2}}$. [3]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (d) Using your answer to part (b), deduce the exact value of $\int_0^{\frac{1}{2}a\sqrt{2}} \frac{x^2}{\sqrt{a^2 - x^2}} \, dx$. [2]

[illegible]

6 The curve C has equation $y = \frac{10+x-2x^2}{2x-3}$.

(a) Find the equations of the asymptotes of C . [3]

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(b) Show that C has no turning points. [3]

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(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

- (d) Sketch the curve with equation $y = \left| \frac{10+x-2x^2}{2x-3} \right|$ and find in exact form the set of values of x for which $\left| \frac{10+x-2x^2}{2x-3} \right| < 4$. [6]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 7 The lines l_1 and l_2 have equations $\mathbf{r} = -5\mathbf{j} + \lambda(5\mathbf{i} + 2\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{j} + \mathbf{k})$ respectively. The plane Π contains l_1 and is parallel to l_2 .

(a) Find the equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

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(b) Find the distance between l_2 and Π . [3]

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- [illegible]

[illegible]

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